

Why pasture growth prediction is difficult

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Abstract

A possible objective of pasture modelling is to make quantitative predictions of pasture growth, for use by farmers in feed budgeting, for example. However, if a model has originally been designed for the purpose of describing or understanding pasture growth processes, it may not be well suited to the purpose of prediction, because (1) it may operate at a different spatial or temporal scale to that required for the prediction, (2) it may not be applicable over the range of situations required for the prediction, (3) it may require input variables that are not readily available or that cannot be measured with sufficient accuracy, or (4) its design may be such that error propagation is not controlled so as to yield sufficiently accurate predictions. This paper discusses the key considerations required for robust pasture growth prediction modelling: prediction specification, model evaluation, choice of explanatory variables, model design, and parameter estimation.

Additional Key Words: pasture growth models, scale, forecasting, explanatory variables, statistics, identification problem, bias, parameter estimation.

Introduction

Attempts to predict pasture growth rate in New Zealand date back at least to the 1950s, when Brougham and Glenday used linear and logistic models to explain the effects of season, weather and species on pasture growth (Brougham, 1956, Brougham, 1959, Glenday, 1959). Later models based on this approach were found to work moderately well at a particular site (Wright and Baars, 1975). Since then, pasture growth models have become ever more complex in an attempt to be applicable across a wider range of situations and break the reliance on site-specific measurements (McCall, 1984, Moore *et al.*, 1997).

Given this history, it seems reasonable for farmers to expect that it would be a straightforward proposition to generate predictions of pasture growth, to help with budgeting of feed supplies for animals, for example. However, the proposition of predicting pasture growth is not as simple as it might first appear. Particular problems are the difficulty in specifying exactly what measure of pasture growth is desired, and then in selecting and then supplying the important explanatory variables that determine the differences in pasture growth between farms, regions and seasons. This paper discusses the key considerations required for robust pasture growth prediction: prediction specification, model evaluation, choice of explanatory variables, model design, and parameter estimation, and highlights the difficulties and errors (conceptual and quantitative)

that may arise, as well as identifying issues that must be addressed to achieve accurate and useful predictions.

Prediction Specification

In providing a prediction, it is first necessary to specify exactly what “pasture growth prediction” is desired. The specification needs to be clear both to the modeller, who must generate the correct information, and to the user, who must know how to interpret the results and use them appropriately in decision making (Campbell, 1999). We begin with two definitions.

“Pasture growth” can have several meanings. Although “pasture growth” technically refers to production of new tissue only (Hodgson, 1979), in common usage “pasture growth” is often used to refer to *net* herbage accumulation, i.e., the difference between new tissue growth and the disappearance of senescent material. The difference between these values can be substantial (Cayley *et al.*, 1980, Chapman *et al.*, 1984), a fact often not acknowledged in reported measurements of net pasture growth. This paper is primarily concerned with prediction of net pasture growth.

“Prediction” need not refer to predicting the future—it simply indicates that we wish to predict what will happen (i.e., what the pasture growth rate will be) in some new situation for which we do not have data. The concept is illustrated diagrammatically in Figure 1. The new situation

might be a new location, a new time period, or a new management regime (e.g., a different grazing or

fertilisation regime), and is specified by means of “explanatory variables”.

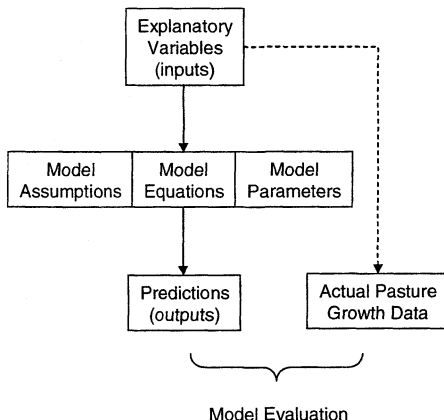


Figure 1. The concept of pasture growth prediction.

The model uses the explanatory variables supplied to calculate a prediction for the new situation.

The main difficulty with specifying what pasture growth prediction is desired is one of spatial and temporal scale. Pasture growth (whether new or net) is highly variable (and discontinuous) in both space and time. Different patterns of variability are observed at different temporal and spatial scales. For example, pasture growth varies with time at the diurnal, weather pattern, and seasonal scales, and with space at the leaf, plant, vegetation-patch, paddock, soil type, farm and topographical scales. Figure 2 illustrates the day-to-day variability of pasture growth rate in a particular paddock over a one-year period. Each data point is already averaged over the area of the paddock and over a time period of 24 hours (c.f., Pearcy *et al.*, 1997). Further averaging over longer time periods, e.g., a month or a year, results in a different value of “pasture growth” at a particular *point* in time, and a different scale of pattern of pasture growth with time. The same is true spatially. In Figure 3 for example, the net pasture growth rates of individual paddocks on a farm range from 35 to 65 kgDM/ha/day. The pasture growth rate averaged over the whole farm is 53 kgDM/ha/d. An important point is that using a representative sub-area (e.g., the paddock with the average pasture mass) to calculate growth rate for a larger area (e.g., a farm) usually results in a

significant overestimate of growth rate (de Wit and van Keulen, 1987, Parsons *et al.*, 2001). In Figure 3, for example, the true farm average growth rate (53 kgDM/ha/d) is much less than the growth rate of the paddock with average herbage mass (62 kgDM/ha/d). Similar variation occurs between patches in a paddock, or between farms in a region.

What temporal and spatial scale to use is determined by the use to which the information is to be put. Two practical examples are: (1) to provide daily whole-paddock predictions for use within a farm system research model (e.g., Sherlock and Bright, 1999), or (2) monthly whole-farm predictions for use by farmers in feed budgeting. A pasture growth prediction, therefore, is not properly specified unless it is clear what area of space and period of time it is averaged over.

Finally in this section, it is important to have some idea of what level of accuracy is required from the prediction. This is determined by the intended use of the prediction, and in particular can be calculated by performing a sensitivity analysis to determine the degree to which errors in the predicted pasture growth rate affect the outcome of the application. For example if the pasture growth prediction is to be used by a farmer in feed budgeting, decisions to conserve or feed herbage may be relatively insensitive to predicted pasture growth rate, as simple management processes often perform

well regardless of the quality of the information used (Warren and Langley, 1999). On the other hand, even quite accurate forecasts may not be enough to

allow the farmer to achieve a significantly improved profitability (Petersen and Fraser, 2001).

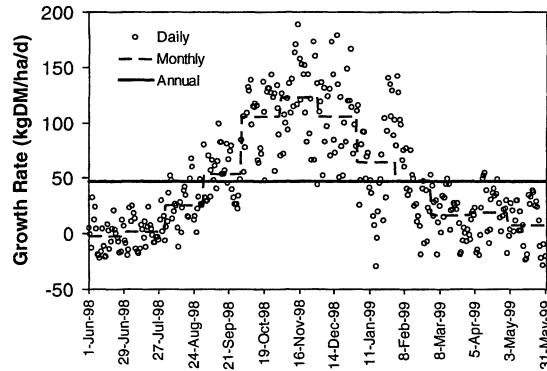


Figure 2. Calculated daily, monthly and annual average net pasture growth rates for a dairy paddock at Ruakura. These data were generated using the MECHANISTIC model.

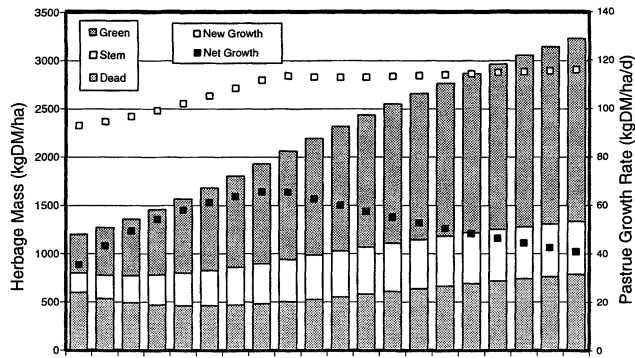


Figure 3. Daily pasture growth rates (new and net, Paddock a/d) calculated for a farm with 20 paddocks of different herbage mass (kgDM/ha). These data were generated using an unpublished pasture model that decomposes whole farm pasture growth rate over a number of paddocks.

Model Evaluation

Closely related to prediction specification is the intended means of model evaluation. This typically involves comparison of model predictions to measured values, usually of pasture mass (usually expressed in kg of dry matter per hectare) (e.g., Riedo *et al.*, 1998). Effective model evaluation

requires a suitable data set against which to test the model, which has been measured at the appropriate scale and sufficiently covers the range of situations over which the model is to be applied (e.g., seasons, regions, pasture types, fertiliser treatments). It also requires an understanding of the measurement errors inherent in the data, which define the limitations of

the evaluation, since evaluating a model against highly uncertain data can provide only a very weak test of model performance.

A useful first step is to perform an evaluation of a “test” model. A suitable test model is any simple (even trivial) model that generates predictions with known properties, whether or not these predictions are accurate. An example would be a model that assumes that pasture growth rate is always 40

kgDM/ha/d. Evaluating the test model (1) helps to clarify thinking as to what the evaluation process is actually evaluating, (2) tends to highlight any serious faults in the data set so that these can be rectified or removed, and (3) provides a benchmark against which the real model performance can be compared.

No single statistic is adequate for evaluating the performance of a model relative to a data set. The key is to assess the model’s performance in a way that is relevant to its intended use. Kendall and Ord

Table 1. Evaluation of pasture growth models using data (N = 4122) from the trial at Dexcel No 2 dairy. MEP = mean error of prediction, MSEP = mean squared error of prediction, MAEP = mean absolute error of prediction, MAPEP = mean absolute percentage error of prediction.

| Model | Description | Explanatory Variables | MEP (kgDM/ha) | MSEP (kgDM/ha) ² | MAEP (kgDM/ha) | MAPEP (%) |
|--------|--|---|------------------|--------------------------------|-------------------|--------------|
| Actual | 1998-2001 growth rates measured for each farmlet | Timing of defoliation, post-grazing masses, farmlet number, month, year | 58 | 0.249x10 ⁶ | 388 | 13.6 |
| | (Woodward, 2001) | Timing of defoliation, post-grazing masses, day of year, weather (daily minimum and maximum temperatures, rainfall, solar radiation, windrun) | -3 | 0.453x10 ⁶ | 524 | 17.7 |

(1990, p.135) propose calculating the mean squared error of prediction (MSEP), mean absolute error of prediction (MAEP) and mean absolute percentage error of prediction (MAPEP), and this approach is also advocated by the French statistician Wallach (Husson *et al.*, 1998). In addition to these, one is usually interested in assessing model bias (systematic deviation from the data), to ensure that the model errors are normally distributed with a mean of zero. More holistic approaches to model evaluation can also be developed, which focus on a wider range of desired model attributes (Rykiel, 1996, Reynolds and Ford, 1999).

Example

In our project, the evaluation data (Figure 4) were calibrated weekly visual pasture mass estimates collected over a three year period (1998-2001) for the 46 paddocks of the Herd 1, 3 and 5 farmlets (2.2, 3.2 and 4.3 cows/ha respectively) at Dexcel’s No 2 dairy farm in Hamilton (Macdonald *et al.*, 2001).

Each farmlet received 200kg of nitrogen in fertiliser per hectare annually. The visual estimation and calibration process resulted in typical data measurement errors of ±300 kgDM/ha (S.L. Woodward, *pers. comm.*).

Based on the visual pasture mass data, monthly average pasture growth rates had also been calculated for each of these farmlets from 1998-2001. These provided a suitable test model with which to develop the model evaluation procedure: the “ACTUAL” model, which used the actual monthly net growth rates measured for each farmlet during 1998-2001. Comparison of this test model with the visual pasture mass data as in Figure 4 highlighted several erroneous pasture mass estimates and missing grazing dates, which were rectified or removed in order to get a “clean” data set, so that deviations between the model and the data were due only to data measurement error and model prediction error.

Table 2. Examples of explanatory variables affecting pasture growth. “Availability” refers to the ease with which this information can be accurately supplied to a model, either by the user (e.g., a farmer), or from historical measurements: high = typically well known, medium = can be measured, low = is difficult to measure.

| Subsystem | Examples of Explanatory Variables | Availability |
|-----------------|-----------------------------------|--------------|
| Site | latitude | high |
| | altitude | high |
| | slope and aspect | medium |
| Soil | soil type | medium |
| | moisture status | low |
| | nutrient status | low |
| | depth | low |
| Pasture | herbage mass | medium |
| | species mix | low |
| | developmental stage | low |
| | morphology | low |
| Management | leaf area | medium |
| | fertiliser | high |
| | seed | high |
| | irrigation | high |
| | cultivation | high |
| Grazing animals | mowing | high |
| | defoliation | high |
| | fouling | low |
| | treading | high |
| Weather | temperature | high |
| | rainfall | medium |
| | solar radiation | medium |
| | sunshine hours | medium |
| Other | wind run | medium |
| | worms | low |
| | pests | low |
| | fungi | low |
| | diseases | low |

The ACTUAL model was evaluated against the data, and prediction statistics calculated (Table 1). There was no significant difference between residuals from different paddocks, and no systematic seasonal bias was evident (Figure 5A). This checked that the residuals were not correlated with any of the input variables, such as time of year.

Sources of Prediction Error

Prediction errors arise from four sources: errors in the explanatory (input) variables, errors in the model structure, errors in the model parameters and computational errors (Wallach and Genard, 1998). We will discuss these in turn.

Explanatory variables

First, care must be taken to select the set of explanatory variables that yield the most accurate

predictions. The variables that explain the differences in pasture growth between different sites and dates include site, soil, pasture, management, animal, weather (historical or hypothetical) and other factors (Table 2). Some pasture models attempt to capture a large number of these factors in order to describe the pasture evolution processes in detail and across a large range of situations. This is an important feature in models that are intended to increase knowledge about the pasture ecosystem (Thornley, 1998).

However, including additional explanatory variables and/or model detail does not necessarily improve a model's predictive ability, because of potential increases in the first three sources of error mentioned above (Reynolds and Acock, 1985, Håkanson, 1995). In particular, explanatory variables are often assumed to have been measured without

error. This is clearly not the case, and these errors can have a major impact on the predictive performance of the model. Furthermore, additional explanatory variables may be expensive or difficult to obtain. Therefore, the choice of explanatory variables must balance (1) the desired range of applicability of the model, (2) the sensitivity of the model predictions to the variables, (3) the relative error associated with measuring the variables, and (4) the ease with which the measurements can be obtained by the user (or otherwise provided by the model, e.g., through look-up tables or databases). These considerations may mean that the best predictions can be achieved by using a relatively simple model requiring a modest number of explanatory variables, which can be easily and accurately supplied.

Model design

Early models used the logistic function to describe pasture growth (Brougham, 1956, Parsons *et al.*, 2001). However, because pasture growth is an emergent property of an extremely complex biophysical system, made up of interacting soil, plant, and animal subsystems (Table 2), this approach has now largely been abandoned in favour of mechanistic models that more closely describe pasture processes (Arnold and Bennett, 1975; McCall, 1984, Moore *et al.*, 1997). Use of mechanism-based models also potentially improves the generality of the model, making it more likely

that it can be used at new sites and in new situations (Dent and Thornton, 1988).

However, this does not mean that detailed mechanistic models are always suited to accurate pasture growth prediction. Simple empirical models (e.g., Glenday, 1959, Wright and Baars, 1975, Cacho, 1993), while not representing as many of the driving processes, could provide more accurate predictions in some circumstances, and may be easier to use. This highlights the need to (1) carefully define what is required from a modelling project, and (2) use the simplest form of model capable of achieving this purpose. In pasture growth prediction projects, what is required is accurate predictions, and an empirical model may be preferred over a mechanistic model if it yields better results (especially if it is easier to use).

One particular challenge of modelling pasture growth is calculating the effects of spatial and temporal variation in pasture mass (Shiyomi *et al.*, 1983) and growth response (Percy *et al.*, 1997) and estimating the impact of this variation on the average growth rate over the area of space and period of time in question. One approach to handling this variation is the use of Gaussian integration (Goudriaan, 1986, Press *et al.*, 1989). This averages over a region by weighting the growth rates calculated at a small number (e.g., three) of points chosen to represent the variation over the region. It is still necessary to specify the variances and covariances in the explanatory variables (Table 2) over the space-time region of interest, which may not be trivial.

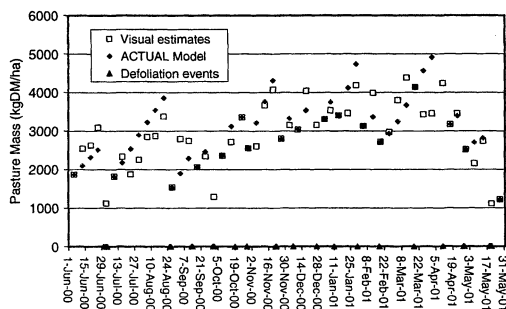


Figure 4. A sample of the calibrated visual pasture mass estimate data from the Dexcel No 2 dairy trial, showing the timing of defoliation events (grazing, cutting), and pasture mass predictions from the ACTUAL model. Data points without a corresponding model prediction indicate estimates made during grazing.

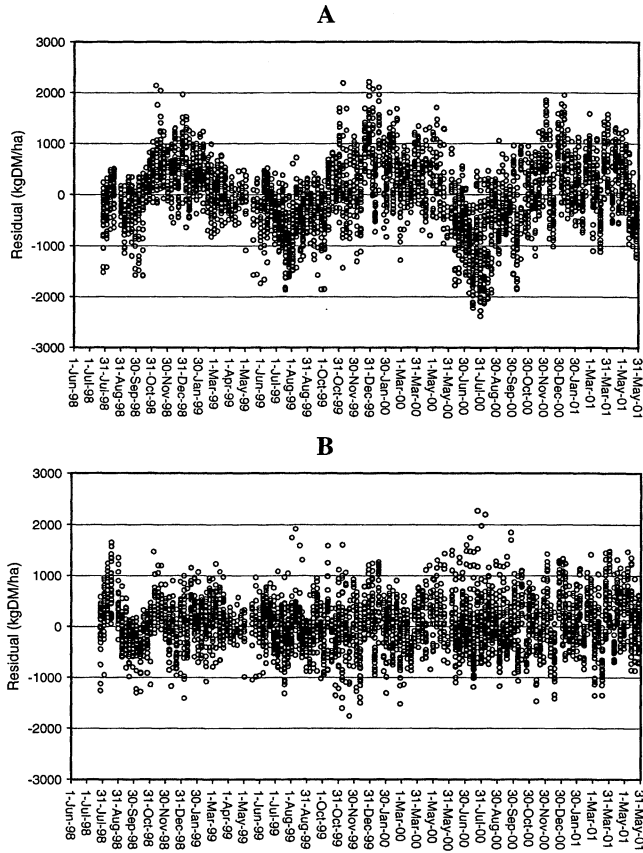


Figure 5. Pattern of residuals from (A) the ACTUAL model and (B) the MECHANISTIC model compared with the Dexcel No 2 dairy data. Note the strong seasonal bias pattern in the residuals from the MECHANISTIC model.

Example

The MECHANISTIC model being developed in our project is described in Woodward (2001). This is a weather-driven dynamical systems model that simulates daily changes in the herbage mass of the vegetative leaf, reproductive leaf, and reproductive stem components of perennial ryegrass, white clover and dead material at a point in space.

The MECHANISTIC model was evaluated against the data set described above, and the results compared with the results from evaluating the test model (Figure 4). The MECHANISTIC model would be unlikely to perform better than the

ACTUAL model, unless it successfully modelled variability not considered by the ACTUAL model (e.g., differences between paddocks). Nevertheless, the MECHANISTIC model has the advantage of being weather- rather than data-driven, and so offers the potential to predict pasture growth at new sites where historical pasture growth rates are not available, and where obtaining them would be an expensive and time-consuming process.

While the predictions from the MECHANISTIC model had only a slightly higher average and percentage error than those from the ACTUAL model (Table 1), the MECHANISTIC

model at its current stage of development has a seasonal bias in its predictions (Figure 5B). This indicates that it does not adequately represent the mechanisms driving seasonal patterns of pasture growth at this site. Further work is required to improve the seasonal behaviour of the model at this site, and also further evaluation against data sets from other sites.

Model assumptions and applicability

Every model is a simplification of the real world, and so includes a number of simplifying assumptions. For example, if the model has been designed for perennial ryegrass-white clover dairy pastures growing on flat, deep soils, which are not nitrogen fertilised (like the MECHANISTIC model), it would not be expected to work well in hill country sheep pastures. By comparison, the models of McCall (1984) and Moore *et al.*, (1997), were designed to apply across a wide range of pasture types. These assumptions must be clearly understood, to ensure that model is used appropriately, or at least is thoroughly tested.

Similarly, when a model is used to produce a prediction, it needs to be stated where this prediction can legitimately be applied. This depends on how widely the model assumptions and the explanatory variables are valid. A powerful and versatile model therefore seeks to reduce the number of assumptions that are made, by including additional explanatory variables to cover a wider range of cases.

Model parameter estimation

Model parameters are determined either from literature or by adjusting them to fit a test data set (Wallach and Genard, 1998). While the latter method does not preclude the model being used for forecasting, determining parameters by adjustment (model fitting) has hidden dangers. First, suitable data must be available for fitting: these data must adequately represent the domain of model application (e.g., a range of sites) and must have well controlled measurement errors (most fitting routines require that the data measurement errors are normally distributed, independent, and have known, and relatively small, standard deviation). Second, even when the data are ideal, fitted parameter estimates may be biased and/or correlated with other parameter estimates (Hopkins and Leipold, 1996). Third, while adjusting parameters may give improved fit to specific data sets, this does not necessarily equate with improved predictive power.

Hopkins and Leipold (1996) have shown that parameter fitting may actually *reduce* the model's predictive accuracy relative to a new set of data. Therefore, automatic parameter fitting should be carried out only with the greatest of caution.

Computational errors

Finally, computational errors arise during the implementation of mathematical models on computer. If the model is defined as a set of coupled differential equations, for example, methods for accurate computer implementation are well established (e.g., Press *et al.*, 1989), and so errors arising from this source should be minimal compared with those mentioned above.

Conclusions

In summary, pasture growth prediction is a complex task, which requires careful specification and statistical rigor. In particular, it is vital that the spatial and temporal scale, the required accuracy and range of applicability of the predictions, and the explanatory variables and assumptions to be used are agreed between the modeller and the user early on in any project, depending on the use for which the predictions are intended.

These factors then determine what kind of model is most appropriate for making the predictions. In many instances, highly detailed mechanistic models designed to describe or understand pasture processes are poorly suited to predicting pasture growth, because of their heavy requirements for data and poor error control. However, purposefully designed mechanistic models can be highly suitable for predictive purposes, provided the errors due to explanatory (input) variables, model form and model parameters are carefully controlled, because they can be made more general than can data-based empirical models.

Finally, good model evaluation requires both a suitable data set that spans the intended range of applicability of the predictions, and a statistically rigorous, well defined process for assessing model performance relative to this data set. The design of this process can be facilitated by evaluation of a test model.

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